ABSTRACT

of the dissertation submitted for the degree of Doctor of Philosophy (PhD) in the educational program 8D07101 – Automation and Control

MARAL TOLYKBAYEVA

ADAPTIVE CONTROL OF A ROBOTIC MANIPULATOR

The main idea of the research. This dissertation develops an adaptive trajectory tracking controller for a serial robot manipulator using a geometric model of its dynamics. The aim is to improve tracking accuracy when the manipulator parameters are uncertain and external disturbances act on the system.

The dissertation proposes and substantiates the following contributions.

– A geometric model of serial manipulator dynamics in which kinetic energy defines the configuration space metric and a coordinate-free equation of motion relates covariant acceleration to joint torques.

– A trajectory tracking control method based on exact feedback linearization of the geometric equation of motion.

– An adaptive robust control algorithm that adjusts the robust correction gain independently for each joint.

The methods were evaluated in MATLAB/Simulink using a two-link planar manipulator model. In the simulations with parameter uncertainty and external disturbances, adaptive gain adjustment reduced the root-mean-square tracking error relative to robust correction with fixed gains.

Keywords: robot manipulator, adaptive control, robust control, trajectory tracking control, nonlinear dynamical systems, geometric control methods, differential geometry, Riemannian manifold, configuration space, metric tensor, feedback linearization, computed-torque control.

The relevance of the research. As robotics applications expand, demands on trajectory tracking accuracy, dynamic performance, and the ability to operate under parameter uncertainty and external disturbances are increasing. Serial robot manipulators are particularly important in industrial automation, medical robotics, robotic materials processing, collaborative robotics, and other fields requiring precise control of motion and interaction with the environment.

Industrial manipulators traditionally rely on position and speed control of electric drives. Advances in electric motors, the development of high-torque actuators, and wider use of direct drives make joint torque control feasible in more applications. Joint torque controllers can directly account for the dynamics of the mechanical structure, regulate interaction with the environment, and implement modern trajectory tracking algorithms.

Joint torque control of a serial manipulator is a challenging engineering problem. The dynamics are nonlinear, link motions are coupled, inertial properties vary with configuration, and model parameters are uncertain. These features limit

the use of conventional linear control methods and call for approaches that account for the manipulator's nonlinear dynamics.

One promising approach is geometric control, which draws on differential geometry, Riemannian manifolds, Lie groups, and Lie algebras. The configuration space of a serial manipulator is modeled as a smooth manifold whose metric structure is determined by the system's kinetic energy. When the kinetic energy quadratic form is nondegenerate, it defines a Riemannian metric that provides an intrinsic description of the manipulator dynamics.

Lie group methods provide a unified way to determine link configurations, velocities, and spatial transformations. The resulting kinematic relations yield an analytical expression for the metric tensor of the configuration space in terms of the system's geometric and inertial parameters. This provides a basis for studying manipulator dynamics and developing methods for transforming the nonlinear model.

Exact feedback linearization provides a basis for trajectory tracking control of a serial manipulator. For a fully actuated manipulator with an exact dynamic model and no external disturbances, a joint torque control law using feedback of generalized coordinates and velocities yields linear tracking error dynamics. The controller gains can then be chosen to set the rate at which the tracking error converges to zero.

The effectiveness of these methods depends on how accurately the model describes the nonlinear dynamics that the controller must compensate for. Masses, moments of inertia, and other dynamic parameters may be known only approximately or may change during operation, for example when the manipulator carries a payload. Adaptive trajectory tracking systems are therefore needed to account for parameter uncertainty and adjust control inputs during operation.

These considerations motivate the development of geometric methods for modeling serial robot manipulators, the investigation of conditions for exact linearization of their nonlinear dynamics, and the design of adaptive control algorithms that improve trajectory tracking accuracy under uncertainty in the manipulator parameters.

This dissertation was carried out under two projects funded by grants from the Science Committee of the Ministry of Science and Higher Education of the Republic of Kazakhstan. Project AP19679327, "Machine Learning Methods for Automatic Control and Inertial Navigation of Mobile Robots," falls within the priority area "Information, Communication and Space Technologies" and the subpriority "Intelligent Robotic Systems." Project AP13068317 is entitled "Development of New Robot Manipulator Control Algorithms for 3D Scanning and Additive Microplasma Coating Deposition."

The object of the research is a serial manipulator with revolute joints controlled by actuator torques.

The subject of the research comprises mathematical models of serial manipulator dynamics, the geometric properties of the configuration space, methods for exact linearization of nonlinear dynamical systems, and methods and algorithms for adaptive trajectory tracking under parameter uncertainty.

The research goal is to investigate geometric methods for modeling and exactly linearizing the dynamics of a serial robot manipulator and to use these methods to design an adaptive control system that tracks prescribed trajectories when the manipulator's dynamic parameters are uncertain.

Research objectives:

1. Analyze existing methods for modeling serial manipulator dynamics, exactly linearizing nonlinear systems, and designing adaptive tracking controllers. Identify their characteristics, advantages, and limitations.
2. Develop a geometric description of serial manipulator kinematics using Lie groups and Lie algebras to determine link configurations and velocities.
3. Represent the manipulator's configuration space as a Riemannian manifold with a metric defined by kinetic energy, and derive analytical expressions for computing the metric tensor from the geometric and inertial properties of the links.
4. Construct a dynamic model of a serial robot manipulator on a Riemannian manifold that accounts for the geometry of its configuration space and the applied control torques.
5. Develop a procedure for exact linearization of the manipulator dynamics through feedback of generalized coordinates and velocities, and use it to design a trajectory tracking control system.
6. Develop an adaptive trajectory tracking system for a serial robot manipulator based on feedback linearization of its nonlinear dynamics. Investigate the conditions for stability and the properties of the tracking error under uncertainty in the dynamic parameters.
7. Simulate the proposed control algorithms, assess tracking accuracy and the effects of parameter uncertainty and external disturbances on control performance, and compare the results.

The main research methods include automatic control theory, Lie group and Lie algebra methods, analytical mechanics, Riemannian geometry, Lyapunov analysis, and computer simulation in MATLAB/Simulink.

Provisions to be defended:

1. A geometric model of serial manipulator dynamics that represents the configuration space as a Riemannian manifold with a metric defined by kinetic energy.
2. A trajectory tracking control method for a serial manipulator based on exact feedback linearization of its nonlinear dynamics.
3. An adaptive robust trajectory tracking algorithm for a serial manipulator that combines exact feedback linearization with independent adjustment of the robust correction gain for each joint. In simulations of a two-link manipulator under parameter uncertainty and external disturbances, the algorithm achieved a lower root-mean-square tracking error than robust correction with fixed gains.

The scientific novelty of the work. A model of serial manipulator dynamics on a Riemannian manifold is presented. The configuration space is treated as a smooth manifold equipped with a metric whose physical interpretation is the manipulator's kinetic energy tensor. For the first time, the manipulator's equation

of motion is formulated in coordinate-free form using covariant differentiation on a Riemannian manifold. Within the proposed model, generalized velocities, forces, and momenta are represented for the first time in a common vector form. Generalized velocity is a tangent vector, and the metric associates the generalized force and momentum covectors with vectors in the same tangent space. In this form, the equation of motion is formally analogous to Newton's second law for a point mass.

An original procedure is proposed for exact linearization of the serial manipulator's dynamic equations expressed in geometric form on a Riemannian manifold. The procedure uses feedback of the manipulator's generalized coordinates and velocities.

A new adaptive robust trajectory tracking method has been developed for a serial manipulator. It combines exact linearization of the geometric equations of motion with adaptive adjustment of the robust correction gain for each joint.

Publications. 8 papers have been published on the dissertation topic. **1** article appeared in *Sensors*, a peer-reviewed journal indexed in Scopus and the Web of Science Core Collection. In 2025, the journal ranked Q1 by CiteScore and Q2 in the *Engineering, Electrical & Electronic* category of Journal Citation Reports. **4** articles appeared in journals recommended by the competent authority for science and higher education in the Republic of Kazakhstan, and **3** papers were published in international conference proceedings. **1** copyright registration certificate was also obtained for a computer program.

The main results of the dissertation work were reported and discussed at 3 international conferences:

1. VIII International Scientific and Technical Conference of Students, Master's Students and Young Scientists, "Creativity of Youth for the Innovative Development of Kazakhstan," April 2022, Ust-Kamenogorsk, Kazakhstan.

2. 18th International Symposium on Applied Informatics and Related Areas (AIS 2023), 16 November 2023, Székesfehérvár, Hungary.

3. X International Scientific and Technical Conference of Students, Master's Students and Young Scientists, "Creativity of Youth for the Innovative Development of Kazakhstan," marking the 125th anniversary of the birth of Kanysh Satpayev, April 2024, Ust-Kamenogorsk, Kazakhstan.

The main scientific results proved in the dissertation, as well as in articles on the research topic, include:

1. A coordinate-free equation of motion has been derived for a serial manipulator. The metric tensor, computed from the kinematic and inertial properties of the links, relates covariant acceleration to generalized actuator torques.

2. A joint torque control law has been derived from the geometric equation of motion. For a fully actuated manipulator with an exact model and no external disturbances, the position and velocity tracking errors have been shown to converge exponentially to zero.

3. An adaptive law has been developed for adjusting the robust correction gain at each joint. It takes into account the position and velocity errors relative to

their specified tolerances and the relationship between the actual and reference time derivatives of the corresponding component of the Lyapunov function.

The dissertation has practical significance. The research results have been incorporated into the 7M07101 Automation and Control program at D. Serikbayev East Kazakhstan Technical University. They are used in teaching the course "Adaptive Methods for Forecasting Process Parameters" as documented in the record of implementation dated 28 February 2024.

For implementation into practice, it is proposed: The computer program was developed by Kadyroldina A.T., Kussaiyn-Murat A.T., and Tolykbayeva M.B. The rights to the program are recorded in the State Register of Rights to Copyrighted Works, as confirmed by Certificate No. 38548 dated 21 August 2023.

The structure and scope of the dissertation. The dissertation comprises an introduction, 4 chapters, a conclusion, a list of 73 references, and two appendices. The dissertation is presented on 132 pages of computer text, includes 53 figures, 11 tables, and 2 appendices.

The introduction explains the relevance of the topic, defines the research goal and objectives, outlines the scientific novelty and practical significance of the work, and states the provisions submitted for defense. Chapter 1, "Analytical Review of Current Research and Theoretical Foundations of Serial Robot Manipulator Control," examines existing approaches to modeling manipulator dynamics and controlling motion, with particular attention to exact linearization and adaptive trajectory tracking. Chapter 2, "Geometric Representation of the Kinematics and Dynamics of a Serial Robot Manipulator," constructs the Riemannian metric of the configuration space from kinetic energy and derives a coordinate-free equation of motion. These results substantiate the first provision concerning the geometric model of manipulator dynamics. Chapter 3, "A Trajectory Tracking Control Method for a Robot Manipulator Based on Exact Linearization," derives a joint torque control law from the equation of motion obtained in Chapter 2. It substantiates the second provision concerning exact linearization of the manipulator dynamics through feedback of generalized coordinates and velocities. Chapter 4, "Investigation of an Adaptive Trajectory Tracking Control System for a Robot Manipulator," augments the control law with a robust correction whose gain is adjusted independently for each joint. Simulations of a two-link manipulator support the third provision concerning the algorithm's effectiveness under parameter uncertainty and external disturbances. The conclusion summarizes the main findings and reports the publications arising from the research and the implementation of its results.